1. (Ungraded) A noiseless 2-kHz channel is sampled every 5 ms. What is the maximum data rate?

1. In this problem, “the channel being sampled” gives us the rate at which the state of the channel can be changed. Thus this refers to the sampling of the analog voltage on the line, not the sampling of some analog real-world quantity like music in order to digitize it! See the lecture slide regarding “digital representation of analog signal representing digital data representing analog information” to understand this better.

A noiseless channel can carry an arbitrarily large amount of information, no matter how frequently or infrequently it is sampled – just send a lot of data per sample. The sampling rate itself hardly matters. In this case, the sampling rate is lower than that which could be achieved on a 2-kHz bandwidth. With that bandwidth, we could afford to inject 2000 samples per second (of course, the receiver would need to sample the channel 4000 times a second, in order to be sure of reconstructing the signal), instead we only use 100 Hz (this is the highest frequency that a receiver sampling rate of 200 times a second can successfully reconstruct). But since we can make each sample carry as many bits as we like, the answer is infinity in either case. It may be appropriate to think of Nyquist and Shannon’s formulae as each being in two parts – the first part provides the number of times the state of the medium can be changed in a second (the number of samples that can be injected into the channel each second), and is $2H$ for Nyquist’s and $H$ for Shannon’s formula. The second part provides the number of bits each sample can be made to carry, and is related to the maximum number of discrete states of the medium that can be reliably recognized at the receiver. In Nyquist’s theorem, the number of such states is assumed to be given, so the bits/sample comes out to be $\log_2(V)$. In Shannon’s theorem, it is related to the SNR, and is $\log_2(1 + S/N)$.

For this problem, the first part (number of samples/second) is explicitly given to be 200, and the second part (bits/sample) is implicitly given to be infinity. The 2 KHz bandwidth is redundant information – it only goes to show that the sampling rate specified is not impossibly high for the medium bandwidth.

2. On a particular physical communication channel, the effects of noise can be ignored if we use no more than 20 different physical symbols (like voltage states, or QAM points). If the bandwidth available is 6 MHz, how many bits/sec can be sent on this channel?

2. Here the effect of SNR has been distilled into a value for $V$ in the question. Using Nyquist’s theorem, we have $2 \times 6000000 \times \log_2 20 \approx 51$ Mbps.

One might raise the question – what is the physical interpretation of using 20 voltage levels? After all, no more than 4 bits can be indicated by them, i.e. only 16 of the 20 states are useful. If you assume that each symbol must be used to transfer only a whole number of bits, then this is true. But otherwise, you can use the extra states over more
than one symbol time to build up information about more bits. The sender and receiver both need to be significantly more complex in this case.

3. A particular modulation system uses 8 distinct symbols, each of the same magnitude of the carrier, but with differing phases; i.e. it uses pure phase shift keying. How will these symbols appear when plotted on a phase-amplitude “constellation diagram” such as in Fig. 2.23 of Tanenbaum’s text (also used in the lecture slides)?

3. By definition, the magnitude of the EM wave for each symbol (i.e. the amplitude) is indicated by the distance from the origin of the corresponding point in the constellation diagram. If each of the 8 symbols have the same magnitude, all 8 points on the constellation diagram will be the same distance from the center, i.e. they will all be on a circle around the center.

Very likely, the 8 points will be equally spaced around the circle, to maximize the distinction between the phases of one symbol and the next.

4. What are the advantages of fiber optics over copper as a transmissions medium? Is there any downside of using fiber optics rather than copper?

4. Fiber has many advantages over copper. It can handle much higher bandwidth than copper. It is not affected by power surges, electromagnetic interference, power failures, or corrosive chemicals in the air. It does not leak light and is quite difficult to tap. Finally, it is thin and lightweight, resulting in much lower installation costs. There are some downsides of using fiber over copper. First, it can be damaged easily by being bent too much. Second, optical communication is unidirectional, thus requiring either two fibers or two frequency bands on one fiber for two-way communication. Finally, fiber interfaces cost more than electrical interfaces.

5. For each of the bitstrings ‘0000000111111111’, ‘0101010101010101’ and ‘0110110111010111’, draw the graph of the voltage for each of the following two methods of injection:
   (a) Direct baseband injection (also known as Non Return to Zero), ‘0’ = +V volts, ‘1’ = -V volts.
   (b) Manchester encoding, ‘0’ = positive transition from -V volts to +V volts at bit center, ‘1’ = negative transition from +V volts to -V volts at bit center.

5. See next page.
6. What is the minimum bandwidth needed to achieve a data rate of $B$ bits/second if the signal is transmitted using (a) NRZ, (b) Manchester encoding?

6. (a) In NRZ, the signal completes a cycle at most every 2 bits (alternating 1s and 0s). So, the minimum bandwidth needed to achieve $B$ bits/sec data rate is $B/2$ Hz.
(b) In Manchester encoding, the signal completes a cycle in every bit, thus requiring at least $B$ Hz to achieve $B$ bits/sec data rate.

7. A cable company provides Internet access over existing cable television cable in a neighborhood consisting of 5000 houses, using the system described in Section 2.8.2 of the text. The company uses a coaxial cable and spectrum allocation allowing 100 Mbps downstream bandwidth per cable. To attract customers, the company decides to guarantee at least 2 Mbps downstream capacity to each house at any time. Describe how the cable company needs to arrange its coax and fiber to provide this guarantee.

7. A 2-Mbps downstream bandwidth guarantee to each house implies at most 50 houses per coaxial cable. Thus, the cable company will need to split up the existing cable into 100 coaxial cables and connect each of them directly to a fiber node.

8. In a typical mobile phone system with hexagonal cells, it is forbidden to reuse a frequency band in an adjacent cell. If 840 distinct frequencies are available, how many of them can be used in a given cell?

8. Each cell has six other adjacent cells, in a hexagonal grid. So it has to avoid any frequency that any of these cells use. This pattern can repeat, and cover the plane endlessly, so each of these six other cells have to avoid each of the others, but no more. The diagram makes this clear, and shows that there are seven “kinds” of cells. For each cell to have the same number of frequencies (so that a user has the same sort of experience no matter what cell they are in), they all have to have the same number of frequencies, which is thus $840/7 = 120$. This is the answer assuming that you hold to Tanenbaum’s assumption that no two cells which are adjacent to the same cell may re-use frequencies.
However, it is also possible to answer the question by repeating a 3-cell pattern without violating the conditions of the question, and this is also acceptable. In that case, the answer is $840/3 = 280$.

The second one saves frequencies, but the first one has the advantage that every frequency is unique to a unique neighbor, as far as a particular tower is concerned. This can facilitate handoff in some situations when two phones are moving into the same cell, and they can be depended upon to have been using different frequencies in their old cells.