Design of Wireless Mesh Networks under the Additive Interference Model

Shu Huang and Rudra Dutta
Computer Science Department, North Carolina State University
shuang5@unity.ncsu.edu, dutta@csc.ncsu.edu

Abstract—Wireless mesh networks are emerging as the next important arena for multihop wireless networking research. Due to several characteristics of these networks, they are amenable to network capacity and resource design in the same manner as more traditional wired networks, with the important difference that wireless interference must be accounted for in design. Studies have already appeared in the literature on such network design. In this paper, we consider this design problem. We mention previous formulations, which address the binary model of interference. We then consider the physical additive interference model, which is the physically more realistic one. So far in literature, the additive nature of interference has been ignored in this context for simplification. We show that existing techniques are not sufficient to address this case, and go on to present a new technique, utilizing blossom inequalities, which can find solutions to this problem. Numerical results show that our approach provides good results in practice.

I. INTRODUCTION

The design of wireless mesh networks is a new field of network design that has seen growing interest in the last few years. Wireless mesh networks are a new area for multihop wireless networking that is confidently expected to be an important entity in the campus- or even metro-scale access networks. These networks exhibit characteristics that are novel in the wireless context, and in many ways more similar to traditional wired networks. Because mesh networks are intended to form a backbone over a significant area, node deployment is planned. Nodes are not mobile, and link quality is reasonably stable, with dependable bandwidth over time. Traffic demands made on the network also are reasonably large, and aggregates are predictable in the same sense as in traditional networks of this scale. These networks thus present a very different picture from mobile ad-hoc wireless networks, and are amenable to network resource design methodologies as traditionally applied to planned network deployments.

However, the one wireless reality that must be accommodated in design is that of interference. It is, of course, possible to assume a CDMA-like environment, and ignore radio interference. However, in the short term, many wireless mesh environments are likely to be built utilizing 802.11 technology and similar, where such an assumption is not realistic. Further, with mesh applications, as opposed to cell telephony where every user node may be expected to communicate only with the tower, the use of multiple radios at each node is fully expected. In such cases, two radios at the same node may not be possible to use simultaneously to transmit and receive because of near-channel interference, if the channels in use are the same or close together. Finally, even transmissions involving completely different node pairs may interfere with each other because of their physical location and the radio medium. The design problem must be formulated with explicit representations of these constraints.

Generally speaking, the design problem can be seen to consist of three subproblems: 1) Routing problem; i.e., how the traffic demands are routed on the multi-hop network. 2) Channel assignment problem; this problem consists of two parts: the connection-radio assignment of radios to connections and the radio-channel assignment of channels to radios. 3) Scheduling problem; since different connections may share the same wireless link, they need to be scheduled. Realistically, the sharing of the link is achieved by time-slooting. Therefore, this subproblem is to assign time slots to connections, on each link.

A. Prior Work

In [8], Kodialam and Nandagopal study the routing and channel assignment problems in single-channel networks based on the protocol interference model. In [9], they extend the work to multi-channel networks and take the scheduling problem into consideration. The authors propose an initial interference model and claim that this model is equivalent to a somewhat simpler model, which is used to derive the main conclusions. However, we find this claim to be incorrect; the initial model is in fact overly restricted. We discuss this further in Section III. However, since the approach in that work is based on the later formulation, this over-restriction does not affect the correctness of any other result of [9].

In [1], the authors studied the same problem as in [9]. The difference is that, in [1], the radios are not allowed to be adjusted during the operation time of the network. In [6], the authors also present an ILP formulation to maximize the throughput. For both the protocol interference model and the physical interference model, conflict graphs are constructed. However, in the physical model, the weights of edges in the conflict graph are not linear.

B. Our Contribution

The formulations for this problem available in the literature may largely be classified as “symmetric”. The physical additive interference model, so far not considered in this literature, is not amenable to the symmetric formulation. We provide an asymmetric formulation that, while more complex, is still...
linear and reflects this interference model accurately. Since the additive interference model is not amenable to symmetric formulations, it is not adequately addressed by the approaches available in the literature. We present an approach that can be used to obtain solutions for the network design problem under this interference model. Our approach is based on the insight that one part of the problem is essentially a generalized matching problem, and iterative use of blossom inequalities can be used to obtain solutions. For the other parts of the problem, we use standard approaches. We obtain numerical results by generating random networks and applying our algorithm. The results show that our approach is of practical value.

The rest of this paper is organized as follows. In the next two sections, we discuss the different ways to formulate the network design problem for WMMs. Section II discusses formulation of all aspects of the problem other than interference, and Section III discusses formulations of interference constraints. In Section IV we present our approach to obtaining a solution to the design problem. Section V presents numerical results and we conclude in Section VI.

II. INTEGER LINEAR PROGRAM FORMULATIONS

In the following subsections, we present an edge-based ILP formulation of the problem, followed by a node-based one. That is, the variables of interest are defined on links in the first case, nodes in the other. All aspects of the problem other than radio interference are represented in these formulations. As we shall see in the next section, the same choice presents itself in formulating interference constraints, so we need to have these alternate formulations for the rest of the problem available.

Formulations for the protocol model (discussed below) are available in the literature, but we define our own below, because we introduce the variables that allow us to go on to formulations of the additive model for which we contribute a new formulation that we can use in our solution approach. The problem is decomposed into the routing problem, channel assignment problem and the scheduling problem. The constraints are ordered by these subproblems. This formulation is similar to the general multi-commodity flow problem in networks. We do not list any specific objective function, this general formulation can carry over easily to different network design objectives.

A. An edge-based problem formulation

We assume that the traffic demands are given as a traffic matrix \( A = [\lambda_{ij}] \), where \( \lambda_{ij} \) is the traffic demands (say, in Mbits/sec) from node \( i \) to node \( j \). As in [9], the physical topology is \( G(V, A, A') \), where \( V \) is the set of wireless nodes, \( A \) is the set of wireless data links and \( A' \) is the set of interference links. Two nodes are connected by a data link if the receiver is within the communication range of the sender. Two nodes are connected by an interference link if the receiver is out of the communication range but within the interference range of the sender, \( i.e., \) a transmission by the sender will be perceived as significantly heightened noise at the receiver, thus degrading the Signal-to-Noise Ratio (SNR) for (therefore, interfering with) a transmission from some other nodes that the receiver might be trying to receive. All links are directed. Let the number of channels be \( N \), the number of radios at node \( a \) be \( K(a) \) and the number of time slots be \( T \). Here, the concept of time slots is used in the same manner as a TDM slotted medium. The problem posed is that of assigning the requisite amount of data transfer to each traffic demand component over the entire \( T \) slots. Let \( C \) be the capacity of a channel, in the same units as the traffic matrix components. We need to find the routing, channel assignment and the scheduling.

We define the following variables. Let \( r^{(ij)}_{ab,n} \) be the traffic flow from \( i \) to \( j \) and traversing link \( ab \), \( r^{(ij)}_{ab} \) be the traffic flow from \( i \) to \( j \) traversing \( ab \) and using channel \( n \). Let \( i_{n,t} \) be binary, which is 1 if the channel \( n \) on link \( ab \) is used in time slot \( t \). Then, we have the following constraints.

- Routing constraint:
  \[
  \sum_{b: ab \in A} r^{(ij)}_{ab} - \sum_{b: ba \in A} r^{(ij)}_{ba} = \begin{cases} 
  \lambda_{ij} & \text{if } a = i \\
  -\lambda_{ij} & \text{if } a = j \\
  0 & \text{otherwise}
  \end{cases} 
  \forall ab \in A, \forall i, j
  \]  
  (1)

- Channel assignment:
  \[
  \sum_{n} r^{(ij)}_{ab,n} = r^{(ij)}_{ab} \forall ab \in A, i j
  \]  
  (2)

- Scheduling:
  \[
  \sum_{ij} r^{(ij)}_{ab,n} \leq C \sum_{t} i_{n,t} \forall ab \in A, n, t
  \]  
  (3)

  \[
  \sum_{b': ab' \in A} \sum_{n} i_{n,t}^{(ab')} + \sum_{b': ab \in A} \sum_{n} i_{n,t}^{(b'a)} \leq K(a) \forall a \in V, t
  \]  
  (4)

Constraint (II-A) ensures the flow conservation. Constraint (2) ensures that the flow is distributed to channels. The scheduling constraints ensure that the time slots are assigned to flow such that the channel capacity and radio availability are respected.

B. A node-based formulation

The edge-based formulation presented above is more common in relevant literature. It maps the traffic flow to channels directly and thus hides the detail of the radio assignment. However, the capability of describing more complicated cases (\( i.e., \) nodes equipped with heterogeneous radios) is lost in consequence. A more explicit option is the node-based formulation that defines variables for radios at each node. As we show later, this model can also naturally apply to the physical interference model. We present such a model below.

We define additional variables as follows. Let \( r^{(ij)}_{ab,k,g} \) be the traffic flow from \( i \) to \( j \), traversing \( ab \) and using radio \( k \) at \( a \), \( g \) at \( b \). \( s_{k,n,t} \) is a binary, which is 1 if the radio \( k \) at node \( a \) is transmitting on channel \( n \) at time slot \( t \), \( t^{(a)}_{k,n,t} \) is a binary,
which is 1 if the radio \( k \) at node \( a \) is receiving on channel \( n \) at time slot \( t \), \( i_{k,g,n,t}^{(ij)} \) is a binary, which is 1 if the radio \( k \) at node \( a \) is transmitting to radio \( g \) at node \( b \) on channel \( n \) over time slot \( t \). Now we have the following constraints:

- **Routing constraint** as same as constraint (II-A).
- **Connection-Radio assignment**

\[
\sum_{k} \sum_{g} r_{ab,k,g}^{(ij)} = r_{ab} \quad \forall ab \in A, ij
\]

(5)

- **Radio-Channel assignment and scheduling**

\[
\sum_{j} r_{ab,k,g}^{(ij)} \leq C \sum_{n} \sum_{t} l_{ab,k,g,n,t}^{(ab)} \leq \sum_{n} s_{k,n,t}^{(a)} + t_{k,n,t}^{(a)} \leq 1 \forall a \in V, k, t
\]

(6)

\[
\sum_{b \in V} \sum_{g} l_{ab,k,g,n,t}^{(ab)} \leq s_{k,n,t}^{(a)} \forall ab \in A, k, n, t
\]

(7)

\[
\sum_{a \in V} \sum_{k} l_{ab,k,g,n,t}^{(ab)} \leq s_{k,n,t}^{(b)} \forall ab \in A, g, n, t
\]

(8)

\[
\text{Constraint (5) ensures the flow from } i \text{ to } j \text{ traversing } ab \text{ is assigned with interface(s). Constraint (6) ensures the capacity of the link is respected. Constraint (7) ensures a radio can be either transmitting or receiving, but not both, on a channel in any time slot. Constraint (8) and (9) ensure that a radio can communicate with at most one radio. In this, we have made the assumption, usual in the WMN context, that most traffic is unicast, and utilization of the broadcast nature of the medium for multicast traffic is not useful.}

**III. FORMULATIONS OF INTERFERENCE MODELS**

In wireless networks, radio interference is an essential characteristic. To clarify the consequences of different decisions in formulating this aspect of the problem, we first discuss the issues to consider in deciding between alternative formulations.

**A. Types of Models**

The interference modeling can be classified as edge-based and node-based, similar to the basic problem. In the edge-based model, the representation of interference is equivalent to a statement such as: “The link \( ab \) will interfere with the link \( cd \) if both of them are active on the same channel at the same time”. Thus links \( ab \) and \( cd \) cannot be scheduled to be active at the same time slot. (Note that this remains true even if \( cd \) does not interfere with \( ab \).) The model is fundamentally symmetric. In the node-based model, the statement of interference constraints take the form: “If node \( A \) is transmitting to \( B \) and node \( C \) is also transmitting, then \( C \) will interfere with the reception of \( B \)’s transmission at \( A \)”. Note that radio interference actually occurs at the receiver, so this model more closely represents the physical reality. However, the model is clearly asymmetric (again, realistic). Naturally, if we choose a node-based formulation for interference, we should correspondingly choose the node-based formulation for the rest of the problem as given in the first formulation of the previous section, and similarly for the edge-based formulations.

The links may be modeled as directed or undirected. If an RTS/CTS/DATA/ACK model of communication is assumed, following several popular wireless LAN protocol models, then each link is constantly being used in both directions, so the links can be naturally considered undirected, and the edge-based formulation is useful. The other alternative is to consider time in finer granularities, and attempt to schedule transmissions from \( a \) to \( b \) separately from transmissions (including ACKs) from \( b \) to \( a \). An edge-based formulation is still possible, but the two directions along each link must be represented by separate variables \( i_{n,t}^{(ab)} \) \( i_{n,t}^{(ba)} \). However, the directed link model can also be used with the node-based asymmetric model (unlike the undirected link model).

Finally, we distinguish between the **additive** and **binary** models of interference. So far, we have presented the binary model (usual in literature), that two transmissions either interfere with each other, or do not. However, the physical reality of radio interference is more complex, and the binary model is a simplified approximation. Realistically, while a receiver \( r \) is attempting to receive a transmission from a transmitter \( t \), transmission by another node \( t’ \) which is close to \( r \) may be strong enough to garble the original transmission; this is adequately represented by the binary model. However, \( t’ \) may be sufficiently far from \( r \) that the effect of its transmission is simply to raise the perceived noise level at \( r \). Whether \( r \) successfully recieves the intended transmission or not depends on the SNR at \( r \). However, this SNR depends not only on \( t’ \), but on every other node in the network transmitting at that time, because the noise effects contributed by them add up. This motivates the more realistic additive model of interference, as presented in [4].

**B. Asymmetric Models**

Symmetric models are available in literature (in particular, above-cited papers), and the interference constraints in such formulations are each of the form:

\[
\sum_{ab \in D_i} l_{n,t}^{(ab)} \leq 1, \quad \forall n, t, D_i \in \mathbb{D}
\]

(10)

where each \( D_i \) is a set of links each of which will interfere with every other. However, each set of nodes \( D_i \) needs to be picked carefully, because the constraint expresses the statement “no more than one of the set \( D_i \) of links may be active at any one time”. As an example of the possible pitfalls in picking \( D_i \), consider the candidate “the set of links that will interfere with some one link \( a'b’ \).” This generates one \( D_i \) for each link \( a'b’ \), \( D(a'b') = \{ab \in A \cup A^t, ab \text{ interferes with } a'b' \} \). These \( D_i \) are used in the initial formulation in [9], and is implicit in the arguments of [7]. Fig. 1 shows an example. In this figure, solid black links are data links and dotted red links are interference links. Suppose there is only a single channel, then it is clear that every other link in the network interferes with the link \( e_A \). (This is because in the protocol model, any
C. Physical Additive Interference Model

We now turn our attention to the more realistic additive interference model. We use the same model as in [4]:

\[
\frac{P_x}{N_a + \sum_{k \neq j} P_k (X(j) - X(i))^\alpha} \geq \beta
\]

(12)

where \(P_x\) is the power of the transmission by node \(x\), \(X(x) - X(y)\) is the distance between node \(x\) and \(y\), \(N_a\) is the noise received by \(i\) and \(\beta\) is the SNR threshold. We assume that all radios transmit at a same given power level. Examining the model, it is clear that we need to use the node-based formulation for the overall problem. Further, no symmetric formulation can address this issue, because the success of a node’s reception depends on the simultaneous behavior of several other nodes. We now translate it into linear constraints using the node formulation as follows, using our asymmetric constraints:

\[
\frac{\left| \sum_{k \neq j} P_k \sum_{q \neq j} P_q (X(q) - X(i))^\alpha \right|}{(X(j) - X(i))^\alpha} + \beta \left( N_a + \sum_{q \neq j} P_q \sum_{k \neq q} (X(q) - X(i))^\alpha \right) \leq \frac{M P_j}{(X(j) - X(i))^\alpha} \forall i, n, t
\]

(13)

Again, \(M\) is a large enough number. This constraint ensures that if node \(j\) is sending data on channel \(n\), then the inequality (12) must be satisfied. Otherwise, because \(M\) is big enough, the constraint is trivially satisfied, and this constraint does not constrain the activity of other nodes.

IV. NETWORK OPTIMIZATION WITH PHYSICAL ADDITIVE INTERFERENCE MODEL

We consider the network design problem that aims at maximizing the concurrent flow, as in [1], [9]. In [9], the authors present a linear relaxation of the constraints and solve the problem using a primal-dual algorithm. Because of the relaxation, the set of constraints becomes a necessary but not sufficient condition. We propose a method that remedies the drawback of relaxation.

Using the notations as above, let the graph be \(G(V, E)\). Two nodes are connected by a link if they are within the transmission range of each other. As mentioned in Section III-C, we do not have interference links in this model because of the additive effect of physical interference. We assume the minimum number of radios at two end nodes of a link may be less than the number of channels available (i.e., \(\min(K(a), K(b)) \leq u(ab)\), where \(u(ab)\) is the number of available channels on link \(ab\)). Let \(T\) be the scheduling length (number of time slots in each schedule period). \(\gamma\) is a scalar of the traffic demands that needs to be maximized. We present a mathematical formulation of the complete problem (we call it Problem I) as follows.
In constraint (18), $\alpha_{ab} = \frac{P_{ab}}{\lambda_{ab}}$, which is the power transmitted from node $a$ and received at node $b$. To prevent a sending node from receiving from other nodes on the same channel at the same time, $\alpha_{aa}$ is $P_a, \forall a$. This also provides an uniform representation of the interference model. Note that if we assume the schedule length $T$ tends to infinity (or equivalently, in a fixed length of schedule, the length of a time slot is infinitesimal), Equation (15) can be changed to

$$\sum_n r_{ab,n} = \frac{C}{T} \sum_t l_{ab,n} \forall a, t$$

In constraint (18), $\alpha_{ab} = \frac{P_{ab}}{\lambda_{ab}}$, which is the power transmitted from node $a$ and received at node $b$. To prevent a sending node from receiving from other nodes on the same channel at the same time, $\alpha_{aa}$ is $P_a, \forall a$. This also provides an uniform representation of the interference model. Note that if we assume the schedule length $T$ tends to infinity (or equivalently, in a fixed length of schedule, the length of a time slot is infinitesimal), Equation (15) can be changed to

$$\sum_n r_{ab,n} = \frac{C}{T} \sum_t l_{ab,n} \forall a, t$$

Leaving, for the moment, the physical interference constraints out of consideration, it is easy to see that this is a capacitated $b$-matching problem. Therefore, by adding blossom inequalities [11], we can obtain a set of necessary and sufficient conditions for the problem without interference. In other words, by solving the linear relaxation problem, we can obtain the fraction of the schedule length that the channel is active on a specific channel such that the constraints imposed by the number of radios at each node and the number of channels on each link are obeyed. In that case, we shall have the following guarantee:

**Theorem 4.1:** Suppose the time any given channel is active on a link, expressed as a fraction of the schedule length, is $a$, and the length of a timeslot similarly represented is $b$, in a solution obtained as above. If $a$ is an integer multiple of $b$, there exists a schedule corresponding to this solution such that the Constraints (17) and (16) are obeyed in every time slot.

**Proof:** Since the polytope described by constraints (21), (22) and blossom inequalities is integral (vertices are 0-1), a feasible solution is a convex combination of the vertices [5]. That is:

$$s = \sum_i a_i v_i, \text{ and } \sum_i a_i = 1, a_i \geq 0 \forall i,$$

where $v_i$ is a vertex. Let $m$ be the number of slots in a schedule length, according to the assumption, $a_i m$ is an integer. Then, a matrix is formed by expanding row $i$ to $a_i m$ copies, for all $i$. Let the transpose of the matrix be $X = [x_{ij}]$, where $x_{ij} = 1$ (respectively, $x_{ij} = 0$) means the link-channel pair $i$ is active (respectively, inactive) at time slot $j$. Then, $X$ is a feasible schedule because of the following reasons. Firstly, each column is a 0-1 vertex, therefore both (21) and (22) are obeyed. Secondly, $XE = s'$, where $E$ is an $m$-vector with every element of the vector equal to $1/m$. Therefore, the fraction of the time that a link-channel pair is active is satisfied.

---

**A. Problem I - Complete Problem**

$$\max \gamma$$

s.t. Equation (14) holds

$$\sum_n \sum_i r_{ab,n} = \frac{C}{T} \sum_t l_{ab,n} \forall a, t$$

**B. Problem II - Resource Allocation to Flows Subproblem**

$$\max \gamma$$

s.t. Equation (14) holds

$$\sum_n \sum_i \sum_b \left( \frac{r_{ab,n}}{C} + \frac{r_{ba,n}}{C} \right) \leq k(a) \forall a$$

$$\sum_n \sum_i \frac{r_{ab,n}}{C} \leq u(ab) \forall ab$$

$$(M - 1)\alpha_{ab} \sum_i r_{ab,n} = \sum_{qm \in E'} \beta_{aq} \frac{r_{ij}}{qm,n}$$

$$\leq C (M\alpha_{ba} - \beta N_0) \forall ba, n$$

The feasible region of problem II is the intersection of three feasible regions, the one described by the flow balance constraints (14), the one described by number of radio and number of channels constraints (21) and (22), and the one described by the physical interference constraint (23). It is interesting to observe that $\sum_n r_{ab,n}/C$ is in fact a linear relaxation of $l_{ab,n}$. Thus, a feasible solution to problem II may be unschedulable. Fig. 2 illustrates an example (we assume interferences do not exist, to study only the effect of the number of radios and channels constraints). There are three nodes in the figure, each node is equipped with one radio, each link has one channel. Obviously, it satisfies the relaxed constraints if each link is active half of the schedule length. However, because of Constraint (17), this is unschedulable. Therefore, it motivates us to add additional constraints such that the feasible region is integral. On the other hand, because the constraints imposed on physical interference have fractional coefficients, this motivates us to relax these constraints.
The blossom inequalities are given as follows:

\[
\sum_{ab \in E(W)} \sum_{n} \sum_{ij} r_{ab,n}^{(ij)} + \sum_{ab \in \delta(W)} \sum_{n} \sum_{ij} r_{ab,n}^{(ij)} \leq C \left[ \sum_{a \in W} k(a) + \sum_{ab \in \delta(W)} u(ab) \right],
\]

\[\forall W \subset V, \text{with} \sum_{a \in W} K(a) + \sum_{ab \in \delta(W)} u(ab) \text{ odd},\]

where \(E(W)\) (respectively, \(\delta(W)\)) is the set of edges with both end-vertices (respectively, exactly one end-vertex) in \(W\). However, the number of the blossom inequalities can be very large. To circumvent this difficulty, we use the separation algorithm developed by Letchford et al [11]. First, we dualize Problem II. Let

\[
\mathcal{L}_{ab,n} = \frac{1}{C} \left[ \sum_{W:a \in E(W)} s_W + \sum_{W:a \in \delta(W)} s_W \right] + \frac{y_a}{k(a)} + \frac{y_b}{k(b)} + z_{ab} \frac{(M - 1) \alpha_{ab} p_{ab,n}}{M \alpha_{ab} - \beta N_a} + \sum_{qm \in E'_{ab}} \beta \alpha_{am} \frac{p_{am,n}}{M \alpha_{am} - \beta N_a}.
\]

where, \(x_{ab}^{(ij)}, y_a, z_{ab}, p_{ab,n}, s_W\) are dual variables to constraints (14), (21), (22), (23) and (24) respectively. We obtain the dual problem:

\[
\min \sum_{a} y_a + \sum_{ab} z_{ab} + \sum_{ab} p_{ab,n} + \sum_{W} s_W
\]

s.t. \(\sum_{ij} (x_{ij}^{(ij)} - x_{j}^{(ij)}) \gamma \lambda_{ij} \geq 1\)

\[
\mathcal{L}_{ab,n} \geq x_{ab}^{(ij)} - x_{a}^{(ij)}
\]

\[
y_a, z_{ab}, p_{ab,n}, s_W \geq 0 \forall a, ab, n, W
\]

It is also interesting to note that, in directed graphs, the blossom inequalities may not exist. However, in our problem, the number of channels on each link and the number of radios on each node are shared by connections of both directions (because the interference model prevents two connections from taking the same channel on the same link), therefore we can simply treat the graph as undirected. The directions of flows will be taken care of by the flow conservation constraints and interference constraints.

Next we describe the algorithm. It merely consists of repeatedly locating violated blossom inequalities using the efficient algorithm from [11], and adding up to a fixed number of them. For the rest of this problem, we simply adopt an existing good approach, which we briefly describe below. The primal-dual algorithm is a fully polynomial time approximation algorithm (FPTAS) that is used in [9]. The analysis follows the work of Garg and Könemann [3]. For the sake of brevity, we first introduce the notation. Let the set of constraints be denoted as \(J\). \(f(ab,n)\) is the flow on the channel \(n\) over link \(ab\). Let \(l\) be a length function. Let \(j \in J\) denote a constraint. Then, constraint \(j\) can be represented as a generalized form

\[
A_j R \leq RHS_j,
\]

where vector \(R = \{\sum_{ij} r_{ab,n}\}\), and \(RHS_j\) is simply the appropriate Right Hand Side of the constraint \(j\). Let \(R_j\) be the set of link-channel pairs on which the constraint \(j\) is imposed. A path \(P\) is formed by a set of link-channel pairs. The maximum amount of flow allowed on path \(P\) with imposed constraint \(j\), \(F(P,j)\), is determined by \(RHS_j(A_j I)^{-1}\), if \(P \cap R_j \neq \emptyset\), where \(I\) is an index vector whose element is 1 if the corresponding link-channel pair is in \(P \cup R_j\). Let \(\delta\) be a predetermined constant. The algorithm is described in table II.

**Algorithm that solves Problem II**

**Initialization:** \(l(j) = 0 \forall j\)

While \(\sum_{j} l(j) \leq 1\)

For \(ij : \lambda_{ij} > 0\)

\(\lambda = \lambda_{ij}\)

While \(\lambda > 0\)

Set weights of \(\mathcal{L}_{ab,n}\) on each link-channel pair (ab,n)

Compute \(P^*\), the shortest path from \(i\) to \(j\)

Let \(u = F(P^*, j)\)

\(\delta = \min\{\lambda, u\}\)

\(\lambda = \lambda - \delta\)

\(f(ab,n) \leftarrow f(ab,n) + \delta \forall (ab,n) \in P^*\)

\(l(j) \leftarrow l(j) \left(1 + \frac{\delta}{f(ab,n)}\right) \forall j : P^* \cap R_j \neq \emptyset\)

End While

End for

\(t \leftarrow t + 1\)

End While

Compute \(\rho = \max_{j} \sum_{(ab,n) \in R_j} f(ab,n)/C\)

Output \(\lambda^* = t/\rho\)

**Table II**

**Primal-Dual Algorithm**

The separation algorithm is described in table III. Given the amount of traffic flow on each link obtained by the primal-dual algorithm, we find a set of nodes such that Constraint (24) is most violated. This is done by forming a cut-tree for the graph with an additional dummy node. Then, the blossom inequality condition is checked on the cut induced by each edge of the cut-tree. For details, the reader is referred to [11]. Because the number of blossom inequalities can be large, a maximum number of iterations is introduced to force the end of the algorithm. However, we see in Section V that introducing only a reasonable number of blossom inequalities usually helps significantly.
If $T$ is infinite, i.e., in a fixed schedule length, the length of a time slot can be infinitesimal, we can set $T$ be the smallest number such that the RHS is integral. That is, we can pick the shortest schedule which will precisely maintain the link allocations as computed. The final result is post-processed if necessary to make it feasible as follows. For each $r_{ab,n}^{(ij)}$, check if constraint (18) is violated or not. If it is violated, find the link $qm \in A \setminus ab$, $l_{n,t}^{(qm)} = 1$ such that $\alpha_{qa} l_{n,t}^{(qm)}$ is maximal. Set $l_{n,t}^{(qm)}$ to 0 and reduce all traffic demands $ij$ traversing $qm$ (i.e., $r_{ab,n}^{(ij)}>0$) by $\delta$ such that constraint (15) is satisfied.

V. NUMERICAL RESULTS

In this section, we present the numerical results. We generate random grid networks. The number of radios at each node and the number of channels on each link are randomly generated. We test our algorithm with various parameters. Specifically, the SNR varies from 4dB to 10dB. The powers sent by all nodes are assumed to be the same, which is varied between 50mW to 100mW. The ambient noise is assumed to be 0mW, that is we isolate the effect of the network nodes only. The traffic matrix is also randomly generated.

First, we evaluate the performance of our algorithm. We obtain a lower bound of the solution by solving the original problem using CPLEX mixed integer programming solver. An upper bound is achieved by solving the linear version of the problem without adding blossom inequalities. The upper bound and the solution given by our algorithm are normalized to the lower bound and displayed in Fig. 3. Because of the computation complexity, the comparison is performed on a grid network that consists of $5 \times 5$ nodes. As expected, our solution falls in the gap of the bounds. In addition, we observe that the gap between the bounds can be large. In Fig. 4, we
keep the distance of each edge in the grid network unchanged while changing \( \alpha \), the power decay factor (i.e., power decays with \( r \) as \( \frac{1}{r} \)). It shows that as \( \alpha \) increases, the maximum objective value also increases, due to the decrease of interference. However, when \( \alpha \) is too large, the network becomes unconnected. The figure also shows that when SNR threshold is larger, the maximum objective value is smaller. This is obvious because a larger SNR threshold implies the node is more sensitive to interference.

The following figure shows how the number of blossom inequalities affects the optimal objective value. We generate grid networks with \( 10 \times 10 \) nodes, and use different number of iterations (that is, the maximum number of blossom inequalities that can be added) to solve each instance. The result shows that when the number of iterations is increased, the optimal objective value is also increased (in most cases). Our experiments show that in every instance the algorithm stops when the maximum number of iterations is reached. It suggests that, if more constraints are added, the optimal objective value of Problem II can be further optimized. However, the incremental benefit is seen to be small. Further, because of the heuristic nature of the Lagrangian relaxation method used to solve the Problem III, the effort taken in solving the previous subproblem may not be worthwhile. Thus we conclude that a reasonable number of blossom inequalities suffices for our approach.

**VI. Conclusion**

We have considered the wireless mesh network design problem under the physical additive interference model, which has so far not been addressed in WMN design. We have presented a new asymmetric model, and an algorithm based on viewing part of the problem can as a generalized matching problem. The performance of our algorithm was validated by numerical experimentations.

**References**


